These are but some of the questions which come to one's mind after attempting to digest this somewhat bewildering review of the subtleties of mental development and child-life that Koffka has managed to compress within the limits of a small book. We have neglected to trace the outline of his treatise, and have omitted many illuminating inferences which carry us far beyond the crude and stodgy interpretations which are still current in present-day American pedagogy. But it seemed more important, in attempting to bring this very significant point of view to the attention of American readers, to devote our space to the new theory and certain of its implications, rather than to review the book with reference to its specific content. We may venture to hope that the volume will be widely read by educational psychologists, because it is filled with nuts for the behaviorist to crack, and if not all are ready to throw over their pet theories of applied psychology in favor of the one here proposed, no one can fail to benefit from the redefinition of his conceptions which the reading of this book will force upon him.

R. M. OGDEN

Cornell University

A Treatise on Probability. By JOHN MAYNARD KEYNES. London, 1921. Pp. xi + 466. Price 18s.

The author of the *Economic Consequences of the Peace* here sets forth a logic of probability, an attempt to treat in comprehensive and systematic fashion the problems of the philosophy of induction with the related general principles of mathematical probability and statistical inference. The work belongs to the course of thought initiated by Leibniz, left undeveloped for a time, brought again into prominence by Hume and Mill, and revivified and applied to the concrete by Laplace and Jevons and Venn. The treatise itself was begun as a fellowship dissertation at Cambridge under the influence of W. E. Johnson, G. E. Moore, and Bertrand Russell, to whom it owes much both for problem and method. After the interruption due to war work, it is now modestly presented "for criticism and enlargement at the hands of others." It intends to supplant by constructive theory some of the negative criticism contained in the chapters on induction in our present-day texts on logic, but it offers also a wealth of critical material to mathematicians who are willing to be concerned with the presuppositions of the theory of probability, as well as a variety of suggestion and caution to all who make use of statistical methods in seeking scientific conclusions.

The book is divided into five parts, with an index, and a selected and briefly annotated bibliography of 25 pages.

Part I gives the general epistemological setting. Probability is defined in its widest sense as a *relation* between propositions (premises) which are derived by direct knowledge, and other propositions (conclusions) which are derived from these indirectly by argument (inference). If not completely implied in and deducible from the data (premises) themselves, all conclusions thus have only a certain probability, and all true induction is effected by the application of a general theory of probability.

Probability, being itself a relation, is always relative to the data upon which it is based. A given conclusion may be probable on one set of data, and improbable on another set. Independent certainty, therefore, can be regarded only as the limit of all possible probability relations, based upon infinite (and therefore practically unattainable) data.

Keynes holds that probabilities, thus generally defined, are not always comparable, even theoretically. Expressed metaphorically, the "path" from data to conclusion is not always straight, and probabilities lying on two different paths may belong to different and incomparable orders. The measurement and mathematical comparison of two probabilities are

possible only when both lie in the same path. This occurs only when the conclusion is one of several exhaustive, exclusive, and equiprobable alternatives. Theoretically such a conclusion exists only for the problems of predicting tosses of a coin, balls drawn from a bag, and the like. The law of non-sufficient reason and the "frequency" theory of probability based upon it are applicable in such cases, but their application to apparently similar practical problems can be made only with reservation and great circumspection. In thus extending their application, the determination of the relevance of data to conclusions is critical. Nevertheless, at least in concrete practical problems where conditions are complex, our judgments of relevance must depend not upon rules, but upon direct insight.

Common uncritical acceptance of the frequency theory has tended to obscure the distinction between the probability and the "weight" of an argument, although it is clear that added evidence, while always increasing weight, may increase, leave unchanged, or actually decrease the probability of a conclusion. The common reliance upon the "probable error," as a measure of weight, is therefore justified only to the extent that further data are more likely to include more relevant and significant data. The probable error, as a partial description of observed facts, must not be confused with its use as an indication of the precision of a generalization regarding all (both observed and not observed) facts of the kind in question.

Part II restates the fundamental concepts and definitions of Part I in terms of symbolic logic. Thus, a/h represents the probability of conclusion a derived from premises h; $a/h=1$ represents certainty; $a/h=0$ represents impossibility, etc. From such simple definitions and axioms, the author develops the "laws of thought" or "necessary inference" (contradiction, excluded middle, etc.) and the theorems of "probable inference" (addition and multiplication of probabilities, inverse probability, etc.)

The remainder of Part II deals with the application of these theorems to various problems. First comes a treatment of the theoretical probability of testimony and the credibility of witnesses. Psychologists and especially those concerned with statistics and with theory of psychophysical method will find a wealth of new and particularly interesting material in Keynes' discussion of the assumptions ("laws of error") which are logically implied in our determinations of the most probable value of variable measurements by the method of least squares, and in our common use of the various averages, the median, etc., as representative values.

Part III contains a discussion of the assumptions which underlie induction, and a logical analysis of inductive generalization. Induction, in its most general form, consists in the observation of a limited number of things, all of which are found to have certain common and therefore essential properties (a known positive analogy), but some of which also are found to have, and others to lack, certain irrelevant properties (a known negative analogy). If we assume that the properties of natural things actually occur in finite "bundles", or are the result of a finite number of "generators," then knowledge of the essential nature of a thing can be increased by seeking and finding additional cases in which new and inessential properties appear. Induction, that is to say, proceeds by increasing the negative analogy, and thus more and more closely limiting the actually essential properties to those of the known positive analogy. Mere multiplication of instances as such would be logically valueless except for the *probability* that the new instances will, as a matter of fact, differ from those already examined in points of unknown but actual positive analogy.

Part IV begins with a discussion of "objective chance" and "randomness." The usual definitions imply the frequency-theory of probability. The author, therefore, redefines the concepts in terms of the general theory previously outlined.

Objective chance does not mean lack of determination,nor does it necessarily indicate a small probability of the occurrence of any given outcome. It exists when we suppose that no increase in our knowledge of the laws of nature and no *practicable* increase in our knowledge of conditions and principles or relevant causal laws will enable us to form more probable conclusions. Random selection exists when, in choosing a member from a group or class, we have no *a priori* warrant for supposing that we shall obtain any one particular member rather than any other, or when such knowledge as we have regarding the particular member obtained is irrelevant to the question whether or not this member actually possesses the characteristic under examination.

The definitions thus arrived at are then applied to a variety of problems: to the question of randomness in the distribution of angular inclinations of the orbits of planets, of random causes for the existence of binary stars, and in the existence of star drifts; to the questions of final causes and argument from design and to the problems of psychical research, etc. Finally comes a discussion of the application of probability to conduct. Here such questions as those of "moral risk" and the "Petersburg paradox" are treated in considerable detail.

Part V is a critical study of the foundations of statistical inference. The first chapter brings out briefly but emphatically the distinction between the purely *descriptive* use of statistics as applied to observed instances, and the far more difficult and uncertain use of statistical methods as a basis of inductive *generalization*. The following chapter gives briefly the historical setting of modern statistics, and deals chiefly with the "law of great numbers" as developed by the theoretical work of Bernoulli and Poisson and supported by the investigations of Quetelet. A great deal of later unjustified application of these principles and theorems, and of the modern preoccupation with the mathematics rather than with the logic of statistics, is to be attributed to the insufficiently critical attitude, the too general claims, and the sometimes unguarded language of these earlier statisticians.

Keynes then considers the conditions which must exist if Bernoulli's and similar theorems are to hold for various theoretical and practical cases. Bernoulli's theorem, for example, is rigorously applicable only if a knowledge of what occurs in early observations will not affect the probability of what will occur in later trials; and, secondly, if these probabilities are all equal among themselves. It can be applied for prediction in 1000 tosses of a coin only if our initial assurance as to the trueness of the coin and the conditions of tossing is so great that 999 "heads" in succession would not alter our half expectation of "tails" on the 1000th toss. "It is, in fact, difficult to give a concrete instance of a case in which the conditions for its application are completely fulfilled." This point is exemplified by an account of the attempts to verify the theorem *a posteriori* by actual coin tossing, dice throwing, lotteries, and roulette. Poisson's theorem accepts the second assumption but not the first, and the still further generalized forms of the theory due to Pearson, Czuber, Tchebycheff, and Simmons, though useful for particular instances, get us really little further in the general treatment of concrete cases.

The author next discusses the problem of the determination of general probabilities from numerical frequencies actually discovered for a group of instances of apparently similar character. He here severely criticizes the lack of logic involved in Laplace's law of succession and shows the absurdities to which its application may lead. The inversion of Bernoulli's theorem is held to be rigorously applicable only if we know that our original data are a true random sample. This knowledge, as already pointed out, is in most concrete cases practically impossible. [Psychologists who find a difference between the means of two *apparently* (but not *certainly*) unselected groups ought not to be surprised if on the next trial with two

apparently similar groups the direction of difference is reversed.] Numbers and mathematical treatment as such can never lead us to truth, apart from consideration of the methods and conditions and more general inductive procedures which we use in obtaining the numerical results. The beginnings of a truly critical work in this connection have been made by Lexis and von Bortkiewicz, a brief account of whose methods concludes the treatise.

It is easy to say in conclusion that logician, mathematician, and man of science owe a debt of gratitude to the author of the treatise for making a first truly comprehensive and critical study of questions which are of fundamental significance to all three disciplines. Not all logicians, of course, will agree that the distinction between knowledge gained by "direct" perception and that gained by argument is logically fundamental. Before we can apply the author's principles of induction to psychical research, for instance, we must have a theory of perception which will enable us to say more definitely whether or not we can directly perceive "spirits", a question which the author's system leaves open. Detailed and critical evaluation of the treatise, however, must be left to those more competent than the present reviewer.

WILLIAM S. FOSTER

University of Minnesota

Education and World Citizenship: An Essay towards a Science of Education. By JAMES CLERK MAXWELL GARNETT. Cambridge University Press, London, 1921. Pp. 515.

Any serious attempt on the part of an English educator to replace the traditional set of mind toward an art of education by a scientific attitude is worthy of notice. When an educator of the prominence of Maxwell Garnett (General Secretary of the League of Nations Union, and Dean of the Faculty of Technology in the Victoria University of Manchester) gives up all his spare time for eight years to the preparation of so pretentious a volume, certainly it is worth more than passing mention. To the Englishman, the book will doubtless mark a significant step toward the recognition of psychology as related to education. To the American, it indicates the systematic and cautious method by which the educators of the mother country are testing materials which, for some years, have been accepted in this country as basic. The book also shows something of the present state of mind of the typical English educator of the more advanced school.

Perhaps the best way in which to contrast the English and the American point of view is to give, without comments, the principal sources of the author's information, and the conclusions he has reached. The book is in three parts, each one of which will be briefly treated.

Book I gives three chapters to a consideration of the aims of education in the past, with a statement of the present situation. Rather startling is the statement that "the most easily observed characteristic of English education at the present time is perhaps its aimlessness." Accordingly, the author discusses a basis for determining a suitable aim toward which a science of education should be directed; Professor John Adams' conclusion that self-realization and a many-sided interest cover the whole field of educational objectives especially appeals to him, and so he finally concludes that the first aim of education during adolescence and maturity must be to build up a "single wide interest."

In Book II (16 chapters) "some of the conceptions of physiological psychology are employed in the attempt to analyse the foundations of character and their effect upon behavior." Rejecting a behavioristic view, and the doctrine of psychophysical parallelism, the author assumes "what Dr. McDougall calls the 'old common-sense view' that psychophysical interaction does in fact take place." Accordingly, in his psychological